

# TEACHING MATHEMATICS IN TIMES OF CRISIS. A CASE STUDY ABOUT DISTANCE LEARNING AT ALMA MATER STUDIORUM - UNIVERSITÀ DI BOLOGNA

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## Abstract

In this article we analyze the strengths and weaknesses of mathematics teaching through Distance Learning in three mathematics courses of the University of Bologna aimed at students of Mathematics, Educational Sciences and Engineering. We interviewed the teachers and invited the students to fill in a questionnaire. We also compared the final marks of the same three courses in the academic years 2018-2019 (traditional classroom teaching) and 2019-2020 (Distance Learning). Finally, we compared the teacher evaluation questionnaires that the students fill out during the year, always in the two cases in question.

Distance Learning brings greater transparency in learning processes than in the classroom. The teachers recognize that they have dedicated more work to preparing the lessons that they can no longer "improvise" on the blackboard. The relationship with formalism also changes, which can no longer be so exasperated and which must be dosed to correspond to different levels of attention. In fact, the students probably have less attention even if they exercise it in greater numbers than in the classroom.

Teachers perception about students is of a greater and more widespread involvement, with more students asking questions and participating (especially among those in some sense weak). In the lessons in which participation tools were offered (chat), there was a more intense and prolonged dialogue over time, even beyond the official lesson time. Conversely, lessons that are totally focused on presentations are more sterile and less profitable: in the face of a top-down approach, students tend to isolate themselves and be passive. An engagement-centred approach instead leads to their unusual participation.

The students for their part declare that they felt more comfortable in a 1-to-1 relationship and that they were more involved and pushed to an active job: distance learning has for some reason increased their sense of responsibility (even if this could be the result of the climate of crisis due to COVID19 and the social distancing that ensued). This has an objective counterpart in the final marks, significantly greater than the previous year.

In the three courses there were cases of students who got involved by proposing exercises and mathematical reflections to the teacher (something particularly exceptional especially in the course of Education), as if the remote context had pushed them to be more involved. Our conjecture is that there is group pressure in the classroom that hinders the expression of some "categories" of students.

Even if we are aware that the results of our investigation cannot be automatically extended to any academic context (not even within the University of Bologna alone), we believe that they can contribute to a reflection on the academic teaching of mathematics both in the remote and in the in the classroom.

Keywords: Mathematics, Engagement, Students and Teachers Well-being.

## 1 INTRODUCTION

March 2020, Italy: academic teaching enters an emergency. Teachers and students find themselves, basically overnight, having to deal with distance learning that completely transforms classic classroom teaching. There are no points of reference that allow them to guide an unexpected situation in terms of methods and timing. There is neither time nor places to discuss how to resume teaching in ways compatible with the lockdown imposed by Covid-19.

Teachers and students lose their daily lives and are forced to give up their routines. The dynamics between teacher and students and within the student group inevitably change.

The reading of what is happening is polarized on two positions: the *traditionalists*, who without a blackboard, classroom and physical presence, feel that what is going on stage is not a real teaching; and the *modernists*, who from platforms, private homes and remote locations, are at ease and consider distance lessons a profitable way of teaching and learning, different from that in the classroom.

These opposing alignments, very present in the narration of distance learning, are obviously a myth. The real positions are instead blurred and return an attitude aware of the complexity of the situation, even if with a slight imbalance of a more positive than negative sign in favor of distance learning. Without wanting to deny the existence of teachers not prepared to properly redefine their teaching, nor of platforms that do not always correspond to needs and habits [4], it is true that in many situations the teachers have shown flexibility, awareness that an evolution is necessary and clarity on the fact that "distance learning should not be thought of as a substitute for what was done in presence" [6].

Compared to other academic teachings, the teaching of mathematics has its own dynamics. In transmitting mathematics, linguistic rigor plays a central role which is expressed mainly in the oral dimension of the lesson. On the other hand, the lesson relies on an important phase of written presentation - typically on the blackboard - in which mathematics is also taught in an almost iconic visual dimension [1]. Finally, the exercises proposed to make students see mathematics "in action" are crucial.

Orality, writing and action through the exercises are three dimensions that change their impact, in different ways and with different effects, in the transition from frontal to distance teaching. All three should be therefore analyzed separately, with greater depth than we do in this article, to investigate the potential and limits of distance learning.

Here we limit ourselves to outlining a first preliminary framework on the basis of the comparison between us co-authors and the available data, provided us directly by the students, i.e. the answers to a questionnaire<sup>1</sup> for the anonymous evaluation of each course institutionally performed by the University of Bologna, and which we considered in the answers given only by the students of the courses of *Mathematics Education* (third year of the Degree course in Mathematics; and fourth year of the Degree course in Education Sciences) and of *Calculus 2* (second year of the course degree in Engineering), in A.Y 2018/19 (lectures in presence) and A.Y 2019/20 (distance learning).

Through these two lenses, we try to ask ourselves whether the experience imposed by Covid during the spring 2020 lockdown could not be a way for network-mediated teaching and learning in mathematics education, as [5] also attempts to investigate. While trying to highlight the limits of distance learning, in the next few paragraphs we will try to support the unexpected strengths that have qualified it in our eyes.

## 2 THE ACTORS OF LEARNING

The subjects of the lesson are three: teacher, student and class. The presence of the class, its internal dynamics and the roles that are created influence the way of being in the classroom of the individual student. The class supports collective learning: a "shy" student is strengthened by the questions of those who come forward, by the nods of agreement from most and by the atmosphere in the classroom. Very soon, however, hierarchies and stereotyped figures are created in a class (physically represented for example by the position in which the different students sit in the classroom itself), and it is not easy for everyone present to get involved in front of the class, responding to stimuli and expressing doubts and questions.

A remote lesson, enjoyed in one's own home environment, through the network and the devices we use, allows for a paradoxically more immediate relationship: it is true that there is an extra technological mean between the student and the teacher, but it is equally true that one important social screen is missing, the class. This activates different relationships. In fact, the class subject almost disappears and the relationship between the individual student and the teacher is strengthened.

From the point of view of the latter, then, the teacher sees (or does not see) every single student in the same way. While it is well known that in the classroom there are "areas" that attract more attention and others that are more camouflaged.

In distance learning all students are on an equal footing, and this frees up the involvement of some, who tend to be peripheral and more passive in presence.

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<sup>1</sup> The questionnaire is divided into two parts: a first quantitative part with closed-ended questions and a second qualitative part in which the student can leave comments.

The professor made sure that everyone was able to follow, despite the drops in attention and connection

The classroom is an ecosystem where different intelligences can intervene [2]. This allows for greater complexity in the explanation and a treatment that is not always linear and clear, but which for this very reason is suitable for engaging the attention of different types of students. In distance teaching, this consolidated and constructive practice becomes an obstacle. The feeling that some get from it is a tiring burden.

*The professor does not follow a linear and clear method of explanation; moreover, she is unable to fully exploit the potential of online lessons, weighing down the lessons.*

Face-to-face teaching is therefore characterized by the ability to manage greater complexity (for example, in the variety of learning styles) but also by the not always positive mediation between teacher, students and class. The distance one is less mediated and allows a more direct and spontaneous one-to-one interaction.

When we go to the classroom, in fact, we also work with a collective dimension - which, however, can make us lose sight of some, or many, individuals - while at a distance we find it difficult to work with the group and gain in direct relationship with individual students.

### **3 A PARTICIPATED AND DEMANDING INVOLVEMENT**

If in the transition from distance to face-to-face teaching the framework of teaching-learning changes, it also changes, as we have seen, the nature of the present subjects (less collectivity and more individuality). Consequently, the relations between them are not, nor can they be, unchanged.

The teacher is required to work for the involvement of students:

*Despite the virtual classroom and therefore the difficulty of establishing a direct relationship, the teacher managed to involve the students in an active and participatory manner. Most of the lessons are not carried out with a frontal and traditional modality, but they are active and innovative. [...]*

Students perceive, when there is, the tension to evolve teaching from traditional methods suitable for the classroom to other more active and participatory ones. And they understand that being involved is a strength in distance learning. Of course, involvement is also crucial when we teach in the classroom, and, in the light of this experiment with reality imposed by Covid-19, we will have to try to understand what has worked remotely and how we can - if we can - bring it into traditional teaching.

What we feel like saying is that it almost seems that face-to-face teaching places the student in a routine that dilutes their participation. Being forced at a distance, having a medium between you and the teacher, pushes you to seek involvement in the first person and consequently to appreciate the efforts in this direction also motivated by the need to overcome the difficulties of a less usual context.

The teacher beyond the screen is under the students' magnifying glass at all times. From a first comparison of the responses of the 2019-2020 and 2018-2019 evaluation questionnaires, it seems to emerge that students evaluate the teacher's work in a much more analytical way. There is a greater expectation on their part about what those who teach must give. In the future it will be necessary to understand if this expectation is the result of the emergency or if it is inherent in distance teaching.

On the one hand, the students observe and demand the teacher's availability, clarity and ability to adapt to the new situation (which, incidentally, is also an indicator of great confidence in the professor's figure).

- The teacher was able to interact very well with the students, giving stimuli, proposing games, lightening these lessons in online mode that are heavy for everyone. Moreover, the teacher has always been very willing to answer any questions we may have and the need for clarification.

The teacher's effort to be clearer and more direct in the explanations is also highlighted by the increase from 56.9% to 79.2% in the percentage<sup>2</sup> of positive answers to the question "Does the teacher explain the topics clearly?"

On the other hand, they are convinced that more involvement and more effort are required from them.

*I think that the teacher [...] was very good at teaching in this way (online), because it is not easy and at least I understood everything, even if at times I struggled.*

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<sup>2</sup> Detected on a sample of 125, in A.Y. 2018/19, and 138, in A.Y. 2019/20, students.

It is as if being in the classroom, to a certain extent, deprives them of responsibility, while being alone in front of the computer or smart phone requires them to play the learning game more personally. Participation in the presence can be a facade, silent, passive. Being on a platform for some reason makes them more active and more demanding.

On the net, then, students feel "strong", as they perceive themselves as more accustomed (or, at least, accustomed to the same extent) to independently enjoy cultural content compared to the teachers themselves. They play at home and relate more equally. They are less willing to suffer the teacher's teaching style and demand standards that they believe they can evaluate and demand.

If this demanding involvement seems evident to us, we must however investigate and understand how many students it belongs to. We seem to be able to conjecture that the transition to distance learning has polarized students, grouping on one hand those who participate and demand quality lessons with high standards, and on the other those who abandon all attempts and leave the course of study completely, hindered from some barrier (technological, family, cultural). Quantifying this second population is crucial to understand to what extent and how to pursue distance learning.

#### 4 THE ORAL DIMENSION OF MATHEMATICS

As we said in the introduction, a mathematical lesson has three characteristic dimensions: oral presentation, visualization through what the teacher writes on the blackboard and action through exercises.

Apparently, the oral component is the one that is least affected by the transition to distance learning: the teacher speaks in the classroom and the teacher speaks on the platform. On the other hand, every exhibition - like every text after all - is built in two: writer and reader, teacher and student [3]. Therefore, the fact that in distance learning there is no direct interaction, mediated by gazes and physical presence, cannot but weigh. Many teachers complain of "speaking for themselves" and this is all the more delicate as the teaching also consists of the construction of a rigorous and exact language like mathematics.

The student is aware of this and, when asked which topics covered in class he understood less, answers:

*Those who are to understand mathematics (operations, formulas, etc.), on the other hand for the situation [in which] we are and to carry out the lesson in an online way that in my opinion is not adequate for this teaching.*

*Or also*

Since we must maintain the centrality of language in teaching mathematics, it is necessary that we rethink oral interactions in a distance lesson. The two risks we run are on one hand to limit ourselves to a communication from one (the teacher) to many (the students). On the other hand, we get stuck in a question and answer with a single student or with a small group. They are two dynamics that bring us back to the hierarchical and stereotyped dynamics in classroom roles. The risk is to disperse the potential for participatory and demanding involvement that distance learning contains in a nutshell.

Of course, it is the teacher's duty to work towards a communication that is as plural as possible and capable of stimulating students' responsibility for learning.

Some of the teacher's practices that seem suitable for stimulating shared exposure with students - and therefore more conducive to receiving the oral component of the lesson - may be:

- interrupt the lesson for about ten minutes, divide the students into small groups that formulate questions to ask the teacher;
- periodically question the class (preferably through a quick online questionnaire) on the solution of the proposed exercises, on the difficulties encountered and so on. It is important that the results are shown in a timely manner so that the individual can rank relative to the class;
- propose mini quizzes (for example, through Kahoot!) to stimulate awareness of key contents;
- strive to ask students direct questions on concepts they would like to assimilate.

In fact, in the classroom and in the presence, the physicality, the looks, the proxemics help to build a dialogue. In distance learning, all this is missing, and we have to find alternative ways to build it .

The hope is that, once we have identified them, we will not forget them when we return to the classroom but we will use them to "hybridize" and make traditional teaching more fertile.

While the involvement with distance learning grows, attention falls further and the practices we indicated - or others developed by the specific teacher - can help keep it alive in a greater number of students. As we will see in a subsequent paragraph, one of the junctions between the two didactics is time management, an issue strictly connected to maintaining attention levels.

## 5 BEHIND THE BLACKBOARD

Perhaps the weakest point in the transition from face-to-face to distance teaching in mathematics is the disappearance of the blackboard.

From the point of view of communication the whiteboard is not a presentation (for example, with PowerPoint). A presentation is made before the lesson, in a different emotional state, without knowing what the climate of the lesson will be. On the blackboard you write during the lesson, while you are doing it and taking into account the atmosphere in the classroom. On the blackboard we write and rewrite. We correct ourselves and overwrite. The gesture with which you write contributes to the lesson: we all have experience of teachers who organize the visible surface in an orderly way and others who trace more "expressionist" signs for us. There are those who use it to convey information, some to convey a meaning, some to visualize a concept.

A presentation lasts over time, it can be used by the teacher in another lesson, or watched once again at a later time by a student who studies. In some sense it is placed in an intermediate position between the oral exposition and the handout or the manual. The blackboard is provisional, it disappears during the lesson to be replaced by other texts, by other contents.

The blackboard is dynamic and, in its own way, interactive and multimedia, if only for the movement that writing live is in the eyes of the students. A presentation is static: there are certainly blackboards that via the web allow us to replicate the dynamism of those in presence, but they are often cumbersome and do not allow the same fluidity to which a mathematician is used.

In distance teaching all this is missing and students require codified, predefined "materials", possibly available before the lesson itself. It is important to provide them, but in the right measure: we must not put too much plaster on the courses we hold or discover the cards on the table. Teaching is also a matter of timing, giving up to provide the materials a priori risks impoverishing our work. It is not a question of "surprise factor" but of direction of the lesson which has its own times and ways which must be the responsibility of the teacher.

Hence, distance teaching requires reflecting and experimenting in two important didactic directions:

- the organization of the lesson, its schedule, the definition of which texts to share before, during and after;
- the technical aspects, the choice of the platform, the software, the consideration of what connectivity the students have or don't have.

The effort of teacher to make the materials available in order to overcome the difficulties that may arise due to the distance is highlighted from the percentage of positive answer to the question "Is the teaching material (indicated and available) adequate for the study of the subject?" that raised from the 51.2% of 2018/19 to 83.3 of 2019/20.

The materiality of the blackboard, when we transpose it into a remote lesson, explodes into a multiplicity of tools and possibilities that must be taken into consideration individually.

## 6 DOING MATH IN AN ACTIVE WAY

In reading the answers given by students with the questionnaire, perhaps the aspect that appears most evolved in distance learning is that of the exercises.

Alongside the traditional exercises, the students account for the proposed educational workshops and problem solving activities.

Exercise, laboratory and problem solving are three different ways of seeing mathematics in action that by becoming part of the lesson can break it by helping to stimulate attention and responding to the participatory involvement of students.

Caution. Students do not always express appreciation for these choices: we note that they found them remarkable, in some cases in the positive, in others in the negative sense. From reading the critical

notations it emerges that workshops and problem solving are experienced negatively because they slow down the course and ask to be an active part.

From our point of view, having different speeds in teaching is instead a tool that meets the needs of different intelligences. And stimulating students to be operationally active means helping them to enter more into mathematics and to assimilate its contents and ways of thinking.

Certainly the exercises proposed by the teacher are essential and appear to be more reassuring, but a teaching that involves students in the phases in which mathematics is done is a teaching that goes deeper.

Educational workshops and problem solving are facilitated by distance learning that overcomes the physical constraints of the classroom - of space, time, mobility and acoustics - and allows greater flexibility in working in groups, each with its own times and channels of communication that do not overlap others disturbing them. In the future, in rethinking teaching, it may make sense to evaluate a mixed organization that provides for these activities to continue to be online.

## 7 TIME AND ATTENTION

Perhaps the greatest difference between face-to-face and distance teaching (not only in mathematics) is time.

Distance lessons must have a shorter duration than the corresponding face-to-face lessons; they must consist of shorter modules and therefore be more fragmented; and must be more frequent. Excessive distancing between one and the other risks making the course more dispersed. Just as explanations that are too "long" are incompatible with the level of attention that students are able to keep.

Students are aware of this.

*Perhaps the online teaching methods have slowed down the course of the lessons a little, because the methods of communication are inevitably different from those in the presence.*

And this should help us predict different times for different stages of a course. From distance teaching we can learn that a class is always an ecosystem rich in biodiversity [2], and that every form of intellectual life needs different times (and ways). A course that keeps a single register fosters one type of intelligence and hinders all others.

The emergency that has unhinged the classes and put us in front of individual students, teaches us to modulate times in a more varied way. Naturally, the return to (also) traditional teaching will allow for some broader moments in which to develop speeches that require important attention. The important thing is that we continue to know that requiring a lot of attention means selecting the students who listen to us; and that it is not appropriate to do it all the time.

## 8 CONCLUSIONS

On one hand, we have not found statements that say that teachers repeat the same type of lessons at a distance as in the classroom, while appreciating the attempt to adapt them to the new context.

On the other hand, the students complain about some technological difficulties.

*The biggest problems were of a technical nature, relating to the functioning of the virtual classrooms. There were several connection problems that caused the professor's voice to jump.*

Just as there are critical opinions on the poor predisposition of teachers to probe and use the potential of the network and the platform.

*fails to fully exploit the potential of online lessons, weighing them down.*

Although it must be said that other students of the same course claim the opposite:

*Among the online courses I followed, I think this is the one that most has been able to adapt to the online teaching method.*

Or who recognize that the difficulties encountered are intrinsic to remote means.

*I found it just a little difficult not to get lost in the various speeches and the various conjectures that we ourselves pupils made, because at times the situation became a bit chaotic, perhaps also due to the means of communication due to distance learning.*

The situation is therefore inevitably blurred and the traditionalist (all face-to-face teaching) and modernist (distance teaching only) positions seem at this point simplistic and a priori.

On the distance learning side there is certainly the social dimension and the construction of a community that remains interconnected over time. Taking face-to-face lessons, being in a group in the classroom, contributes to building the "mathematical community" and cannot be a goal dismissed as inessential.

On the other hand, however, there is that the Covid-19 emergency has led us, in adapting to the limits imposed by platforms and distance learning, to test new teaching methods or, more realistically, to experiment with forms that were in the classroom present sporadically if not in a nutshell.

The equilibrium to which we will have to try to strive is therefore a fair means that takes into account how we have, evolutionarily, adapted to the new situation, also in consideration of the fact that with climate change - of which pandemics are a corollary - sooner or later we will have to deal with other emergencies and having a flexible teaching model capable of adapting to different situations will not be a small resource.

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